Notes by
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অংক লেখার কায়দা:

Problem 1: I saw a girl with a telescope.

(ছবি)

I went to a college with a bag.

Note: We want to avoid confusion.

Problem 2: "Roll no. 1 is not a boy." means

Roll no. 1 is a girl. $\boxed{\text{OK}}$

Everybody in this class is not a boy. means

" " " " is a girl

not a boy means a girl ?? sometimes yes.

" no.

Defn of set; subset*: (see later)

Grammer of Mathematics:

1. $\in$ belongs to or in $1 \in \{1,4,5\}$, $2 \notin \{1,4,5\}$
   $\{1\} \subseteq \{1,4,5\}$
2. $\forall$ for all
3. $\exists$ there exists
4. $\Rightarrow$ implies or only if
5. $\Leftarrow$ implied by or if
6. $\Leftrightarrow$ iff or if and only if
7. $\neg$ not

(2) All men are ~~st~~strong. $\forall m \in MAN$, m is strong.

All girls love dogs. $\forall w \in GIRL$, $\forall d \in DOG$, w loves d.

(ছবি)

③ there exists means there exists atleast one.

Suppose A contains integers less than 10. Then $\exists\, x \in A;\ x < 10$

$\exists\, w \in GIRL \quad \exists\, d \in DOG \quad w$ loves d (ছবি)

<u>Note:</u> • After $\forall$ or $\exists$ we must have a variable.

$\forall a\ ,\ \exists b\ ,\ \forall r\ ,\ \exists z$ etc.

• No formula after $\forall$ or $\exists$.

$\forall n$ even, $n+1$ is odd ✓

$\forall 2n \in EVEN,\ 2n+1 \in ODD$ ✗

<u>Use of $\forall$ and $\exists$ together:</u>

$\forall\, s \in STUDENT \quad \exists\, p \in PEN\ ;\ s$ has p.

For all student s, there exists a pen p such that the student s has a pen p.

<u>Note:</u> some s may have more than one pen.

<u>30/11/23</u> Different s may have the same pen p.

Difference between

$\forall\, s \in STUDENT \quad \exists\, p \in PEN;\ s$ has p

$\exists\, p \in PEN \quad \forall\, s \in STUDENT;\ s$ has p.

④ $(x > 2 \Rightarrow x > 1)$

For any x, if x is greater than 2 then x is greater than 1.

কি হলো $\Rightarrow$ কি হবে.

(5) for [illegible] $\Leftarrow$ for [illegible]

$x > 2 \Leftarrow x > 5$

x is greater than 2 whenever x is greater than 5.
~~only~~ if

(6) Suppose A contains 1, 4, 9, 16. $\exists \sqrt{x}$

$\forall x \in A$ (x is ~~e~~ non negative iff x has a square root)

For all x in A x is non negative if and only if x has a square root.

<u>Note:</u> $A \Rightarrow B$ A only if B A is sufficient for B

$A \Leftarrow B$ A if B B is necessary for A

<u>Converse statement</u>

Converse of $A \Rightarrow B$ is $A \Leftarrow B$.

Converse of smoking $\Rightarrow$ Cancer is smoking $\Leftarrow$ cancer

<u>Ex:</u> find converse and state if true or false.

(1) All fathers are men.
(2) All square numbers are nonnegative.
(3) A number is divisible by 2 and 3 implies the number is divisible by 6.

(7) $\neg$ (I am hungry) is (I am not hungry)

Negation means compliment.

$\neg$ (x is even) is (x is odd).

Note: $\neg\neg A$ is A.

(1) I always speak the truth $\xrightarrow{\neg}$ I lied at least once.

Example: (2) $\forall g \in GIRL \ \exists d \in DOG$; g loves d.

Negation is $\exists g \in GIRL \ \forall d \in DOG$; g does not love d.

Exercise: $\forall a \in A \ \exists b \in B$; 3 divides $a-b$.

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Example: $\neg(A \& B) = \neg A$ or $\neg B$

$\neg$ (Roll no. 1 is not a girl with long hair)

= $\neg$ (girl & long hair) = $\neg$(girl) or $\neg$ (long hair)

Similarly $\neg(A \text{ or } B) = \neg A$ and $\neg B$.

$\neg$(seat reserved for girl or senior citizen)

= $\neg$ (girl) and $\neg$ (citizen).

• $\neg(A \Rightarrow B)$ = A and $\neg B$.

$\neg$ (advance money $\Rightarrow$ take good) = advance money and $\neg$(good taken)

De Morgan's Law: For any two sets A and B, we have

$$(A \cap B)^c = A^c \cup B^c$$

$$(A \cup B)^c = A^c \cap B^c$$

Corollary: $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$

$(A \cap B \cap C)^c = A^c \cup B^c \cup C^c$

Collection of sets:

$\{A\}$, $\{A, B\}$, $\{A_1, A_2, A_3\}$, $\{A_1, A_2, A_3, \cdots\}$

$= \{A_i : i = 1,2,3\}$ $= \{A_i : i \in \mathbb{N}\}$

For an index set I, $\{A_i : i \in I\}$.

Cartesian product:

Relation: Equivalence relation. partition.

$x \rho y \Rightarrow y \rho x$

Function or mapping:

ρ

reflex, symm, trans

reflex, antisym, trans.

$\Downarrow$

$x \rho y,\ y \rho x \Rightarrow x = y.$

Poset: Zorn's lemma

Permutation & combination

If every chain has an upper bound in a Poset P, then P contains atleast one maximal element.

Partition, pigeonhole principle,

inclusion exclusion principle.

• Vacously true: Let $A = \phi$. Then $\forall a \in A$, $a > 0$.

(why ??) ① no element to check!

② If $(\forall a \in A, a > 0)$ is false then,

$\neg(\forall a \in A, a > 0)$ is true statement

$\parallel$

$(\exists a \in A ; a \leq 0)$ but $\nexists a \in A$.

Subset: $A \subseteq B$ means $\forall x \in A$, $x \in B$.

Check: $\phi \subseteq \{1, 2, 3\}$.

• How to proof a statement:

① Direct proof of $(A \Rightarrow B)$. Assume A and proceed towards B.

② Proof by contradiction. Assume $\neg(A \Rightarrow B)$ and ~~proceed towards~~ $\neg A$. find contradiction.

Proof of (n^2 is odd $\Rightarrow$ n is odd)

Sets: $\mathbb{R}, \mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R} \setminus \mathbb{Q}$ $\qquad$ $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

$2\mathbb{Z} =$ $\qquad$, $\mathbb{Z} - 2\mathbb{Z} =$

Ex: $5 \in \{n^2 - 1 : n \in \mathbb{N}\}$

Examples: $\{\frac{1}{n} : n \in \mathbb{N}\}$, $[0,1] =$ $\qquad$, $(-\infty, 3) = \{ \quad \}$

Set Arithmetic $\quad A, B, A^c, \quad A \cup B, A \cap B$ with example,

$A + B = \{a + b : a \in A, b \in B\}$.

$A = \{1, 2\}$, $B = \{25, 27\}$, $A + B = \{ \qquad \}$

$A = (0, 1)$, $B = (-1, 0)$, $A + B =$